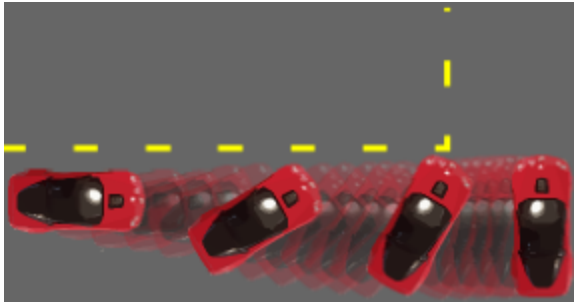
Need for Speed Tokyo Drift: Hotwheels Edition (A Drifting Control Case Study)

Eric Wong and Frederick Chen



Motivation

Pop culture

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Drifting competitions

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Adverse conditions
(snow, rain)

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Goal: Understand and create models that work when traction is lost

Literature Survey

Stability control

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- Liebemann et al.
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- Eyisi et al., 2013 (adaptive cruise control)
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Simulation-based drifting

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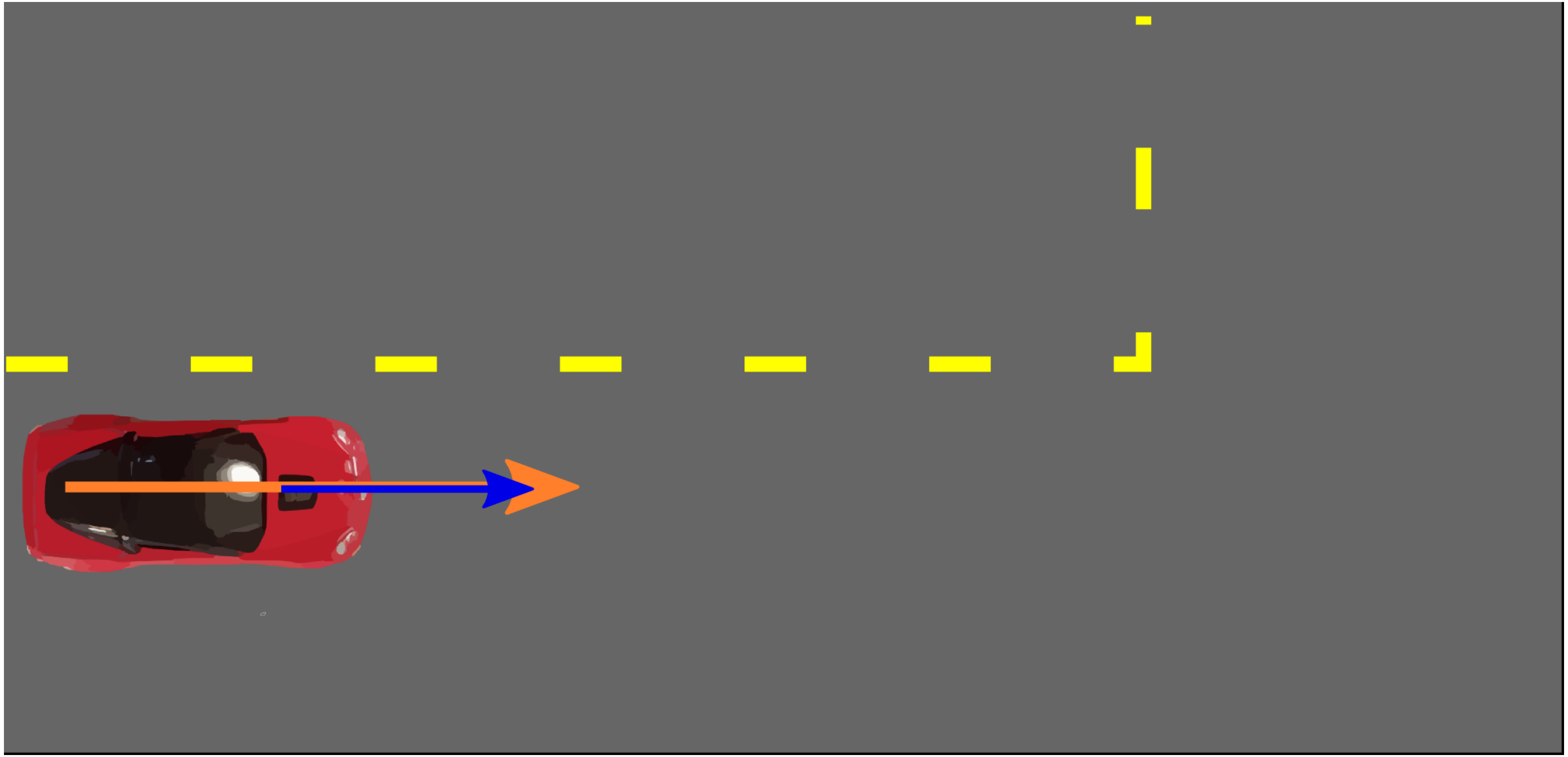
Simulations don’t prove reliability of the system!

Research Questions

1. Can we make and formally verify a reliable controller that safely drifts to the desired range direction?

2. How close can we get to the desired direction?

Drifting motion



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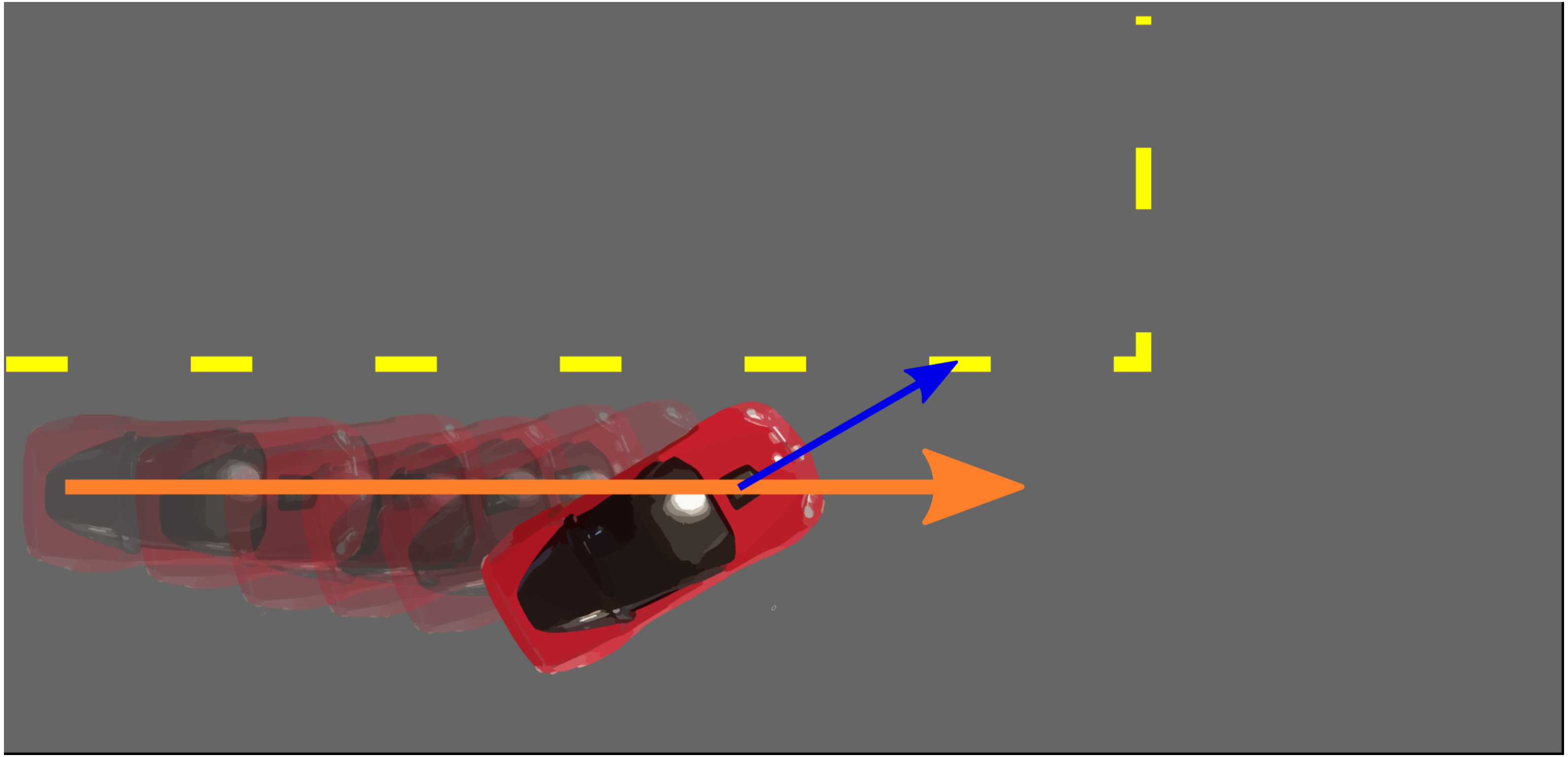

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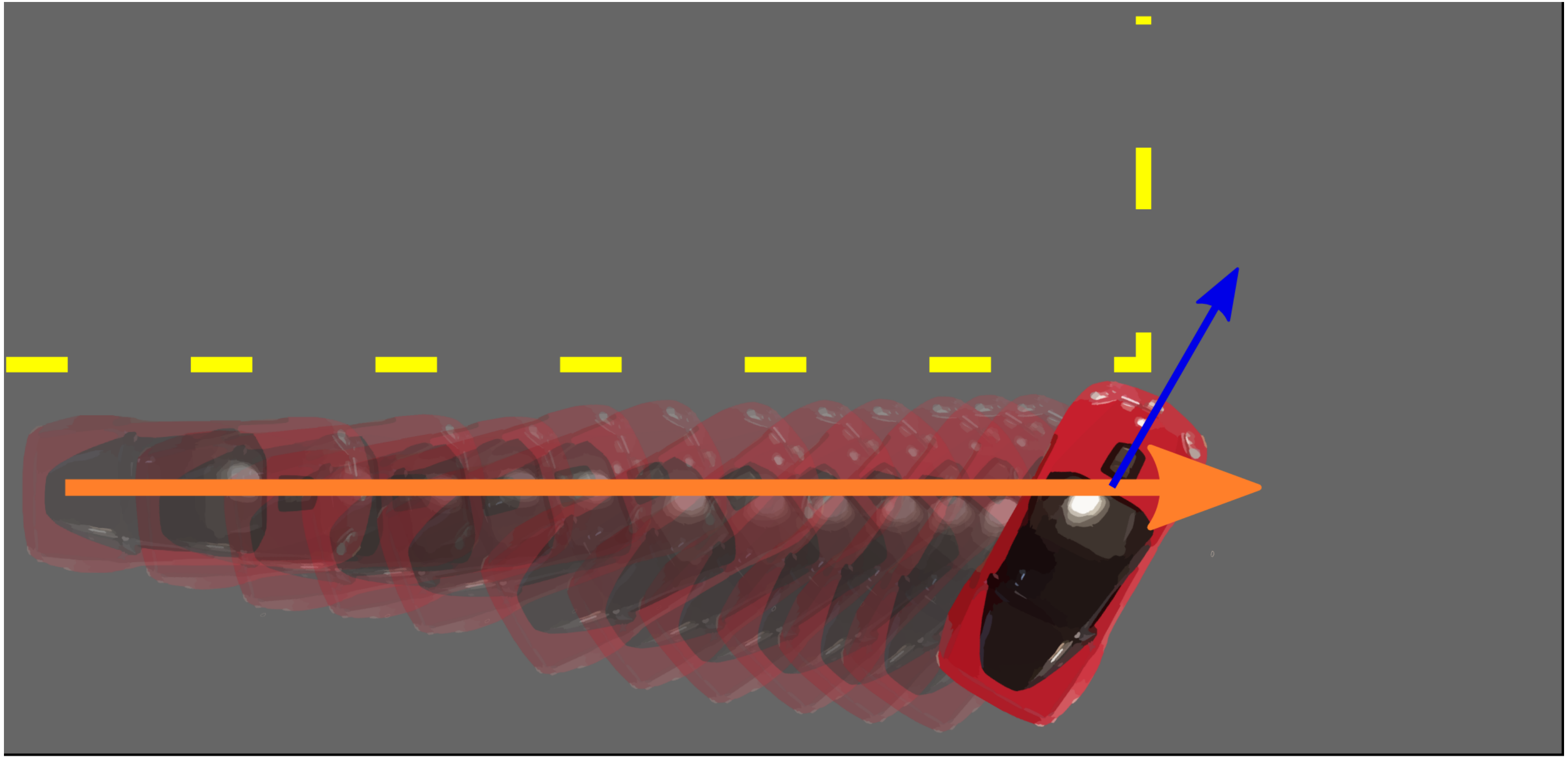

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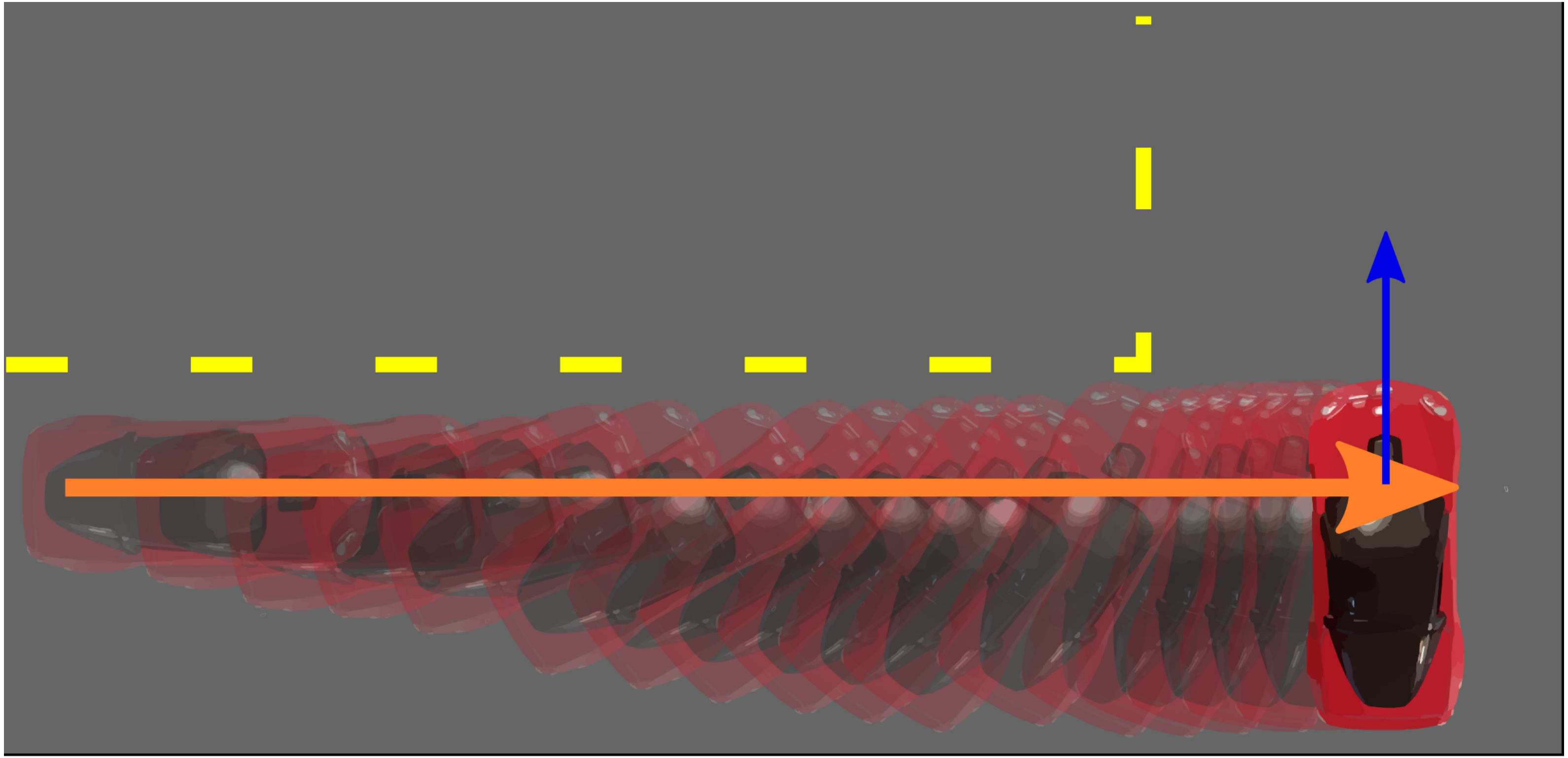
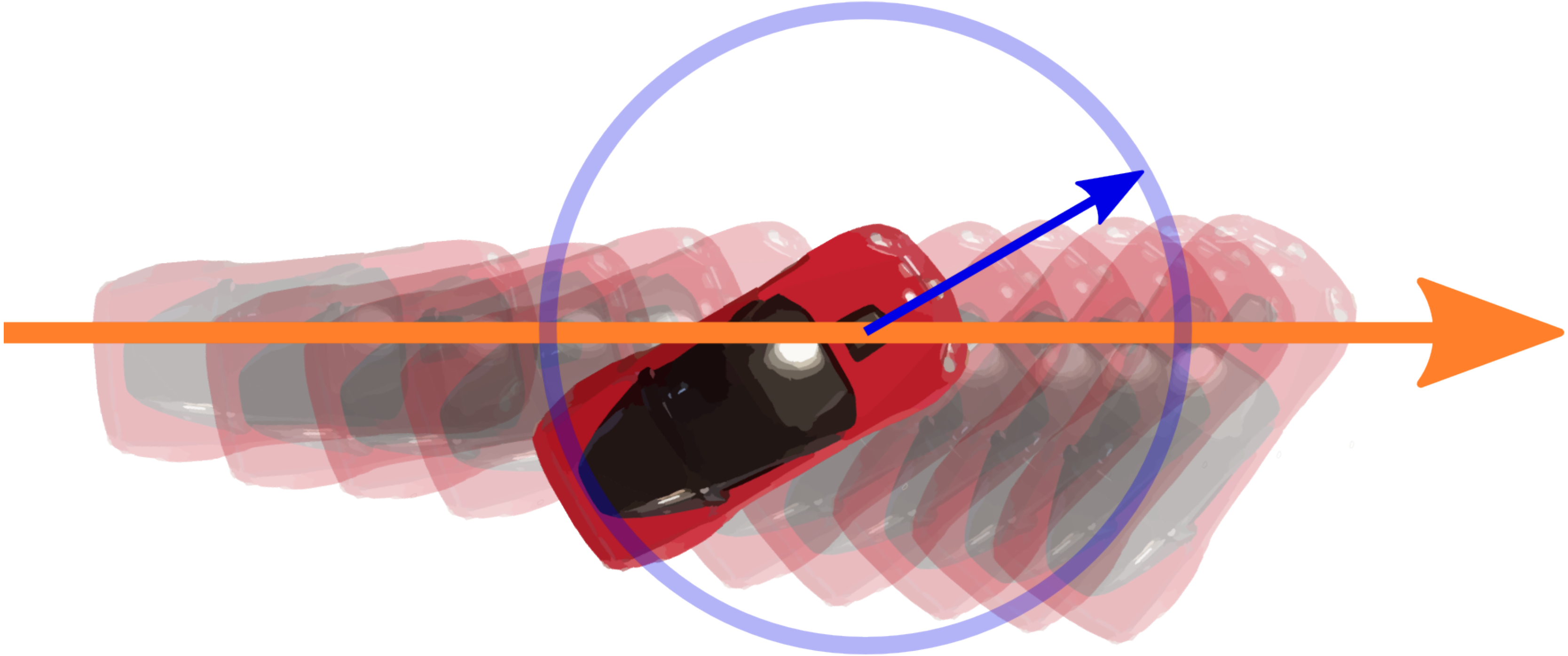


Linear + Circular Motion



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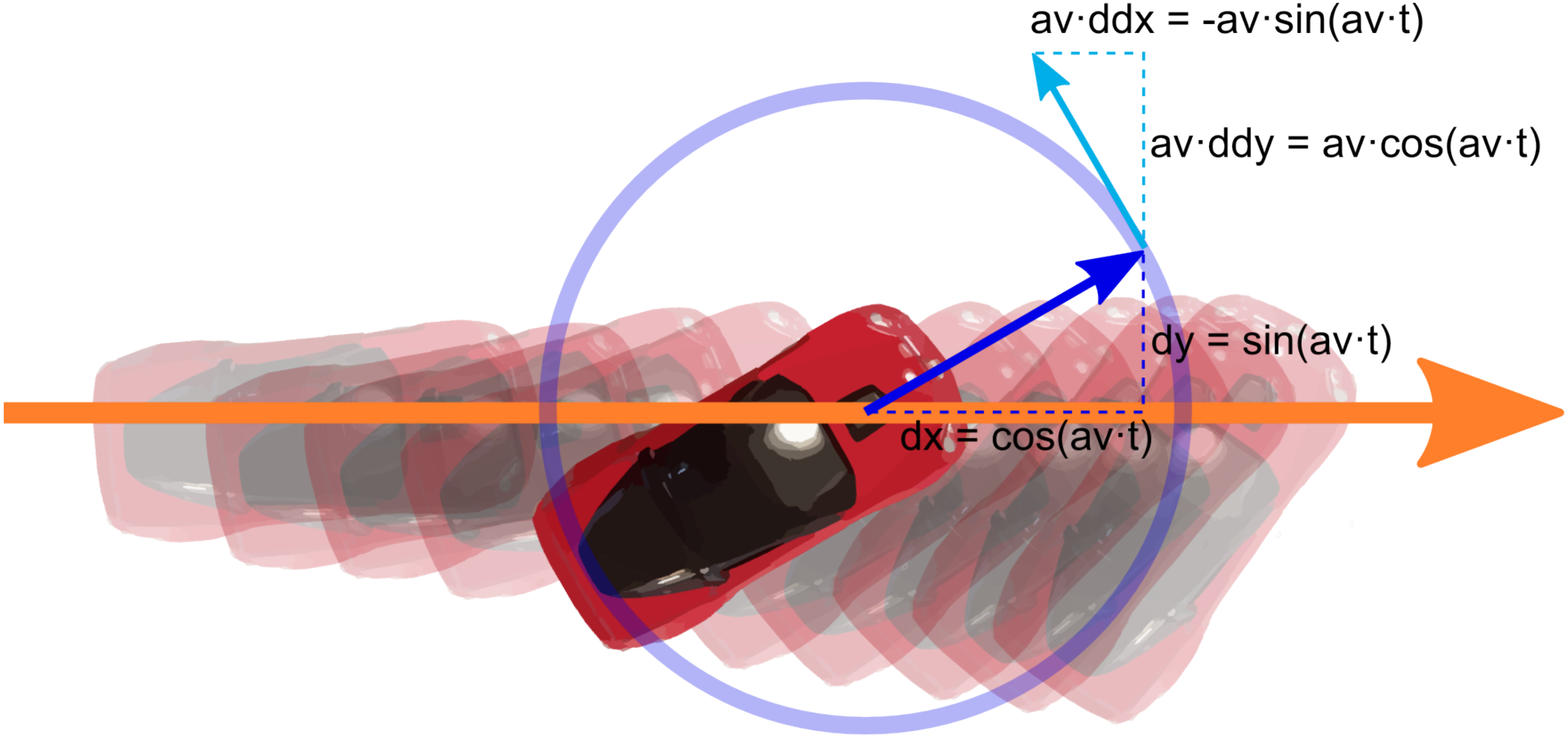

Model Differential Equations

linear motion

$$dyn \equiv (pos' = vel, vel' = acc, t' = 1$$

Model Differential Equations

circular motion on unit circle

linear motion

$$\begin{aligned} dyn \equiv (&pos' = vel, vel' = acc, t' = 1 \\ &dx' = av \cdot ddx, dy' = av \cdot ddy, \\ &ddx' = -ddy \cdot av, ddy' = ddx \cdot av \ \& \ vel \geq 0) \end{aligned}$$

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angular velocity (turning rate)

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angular velocity (turning rate)

We use KeYmaera, a hybrid verification tool for hybrid systems that supports differential dynamic logic to model and prove our properties

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Controller decision

How fast should we turn in order for dx to land in the interval (dx_l, dx_u)?

Taylor Series Bounds

Taylor series bounds provide provable differential invariants

$$dx \leq 1$$
$$dy \leq (av \cdot t)$$
$$dx \geq 1 - \frac{(av \cdot t)^2}{2}$$
$$dy \geq (av \cdot t) - \frac{(av \cdot t)^3}{6}$$
$$dx \leq 1 - \frac{(av \cdot t)^2}{2} + \frac{(av \cdot t)^4}{24}$$

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$$dx_l \leq 1 - \frac{(av \cdot t)^2}{2} \leq dx \leq 1 - \frac{(av \cdot t)^2}{2} + \frac{(av \cdot t)^4}{24} \leq dx_u$$

Use these bounds to find a good angular velocity

Our Controller Guarantees

1. Our controller is guaranteed to not drift off the road

2. Our controller is guaranteed to drift to a direction with within an arbitrary range (dx_l, dx_u)

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Additional Assumption: (dx_l, dx_u) must satisfy

$$1 + 4 \cdot dx_l + dx_l^2 \leq 6 \cdot dx_u$$

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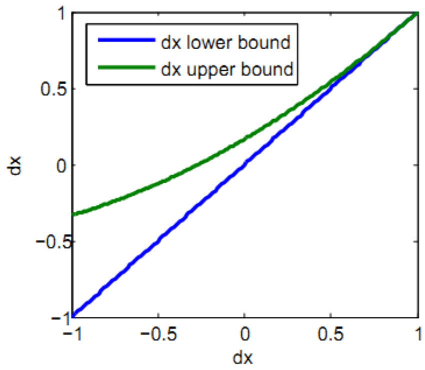
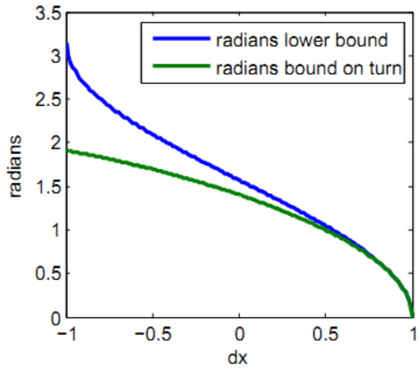
The minimum range enforced by this condition is reasonably small

dxl = -0.5 : 23.19 degrees

dxl = 0.0 : 9.78 degrees

dxl = 0.5 : 2.89 degrees





Increasing the order of the Taylor series approximation relaxes this constraint, feasible up to order 8, due to closed form solutions for degree 4 polynomials

Conclusions

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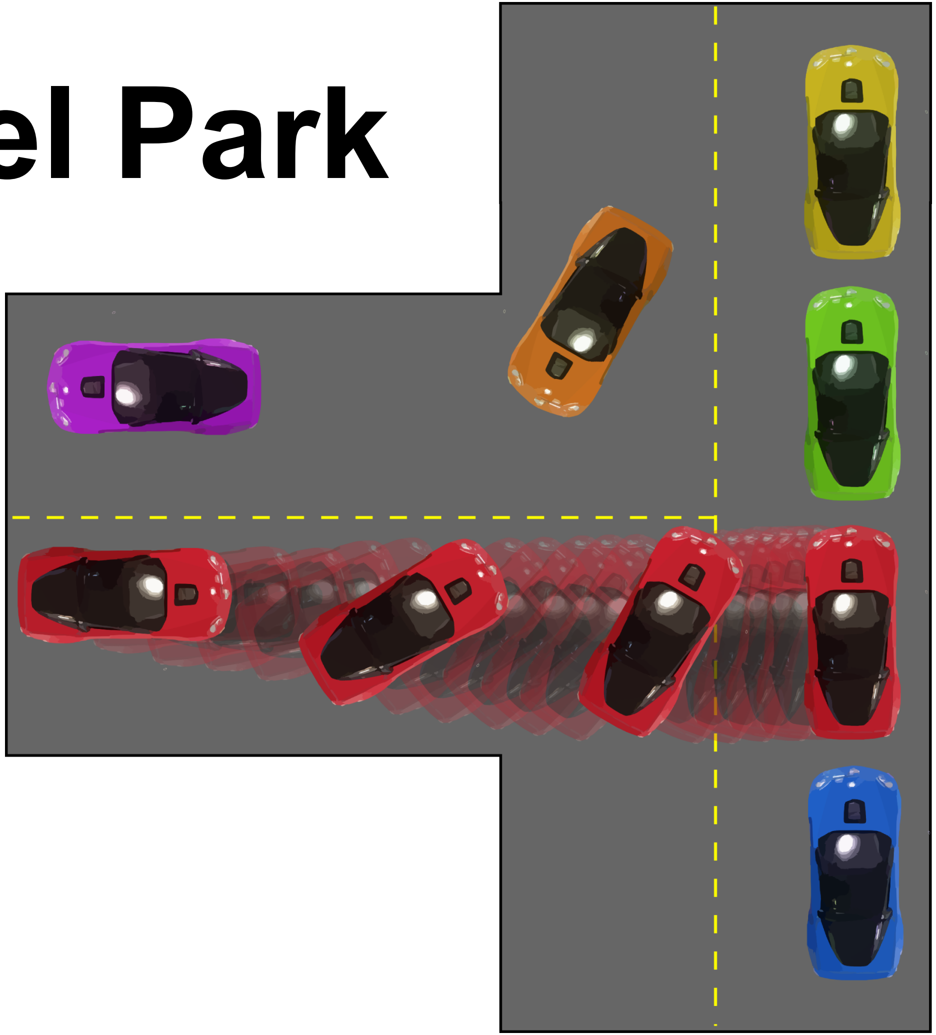
Using a 4th order Taylor approximation our controller can get reasonably small intervals of desired turn, with the potential to go up to an 8th order approximation if necessary

Parallel Park



Future Work

Additional Variables to
Closer Model Reality

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Planning for Unexpected Loss of Traction

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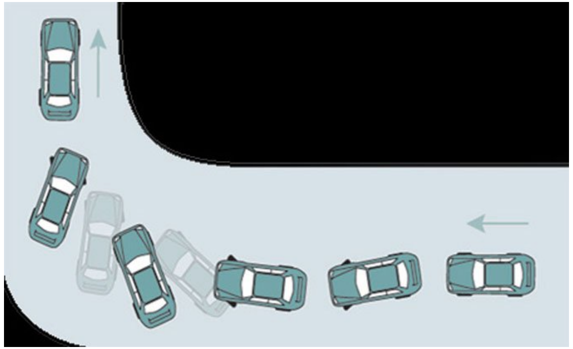
Acceleration while drifting



Questions?